ANALYSIS OF TRANSMISSION LINE MODELS WITH NON-LINEAR ELEMENTS FOR OPPOSITE PHASE AND GROUP VELOCITIES

Synopsis submitted in fulfillment of the requirements for the Degree of

DOCTOR OF PHILOSOPHY

By

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1. Introduction:

Transmission line theory [1] is extensively used in microwave engineering. The main difference between the transmission (Tx) line theory and circuit theory is electrical size. Circuit analysis assumes that the physical dimensions of the device are much smaller than the electrical wavelength, while transmission lines may be a considerable fraction of wavelength, or many wave lengths, in size [2]. Thus the transmission line is a distributed-parameter network where voltages and currents vary in magnitude and phase over its length.

In many ways the transmission line theory bridges the gap between the field analysis and basic circuit theory. With the help of transmission line theory, it is easy to analyze the wave propagation through the medium without exactly solving the Maxwell's equations [3]. Wave phenomenon such as reflection and transmission can be analyzed in circuit quantities (namely voltage and current) rather than in filed quantities (namely electric field intensity and magnetic field intensities).

A two-wire transmission line is schematically represented as shown on Fig.1 A short piece of line length dz can be represented in loss less case as with lumped-element circuit model as shown in Fig.2 The time-domain Tx line equation (also known as Telegrapher's equations for this differential length element are obtained by using the Kirchhoff's current law and Kirchhoff's voltage law [4].

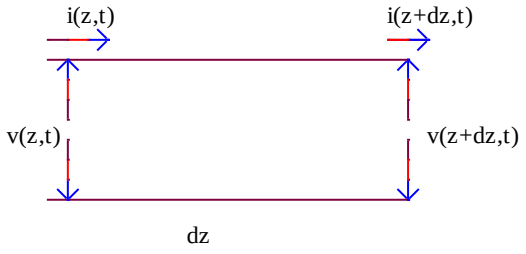

Fig.1 Differential transmission line of length dz.

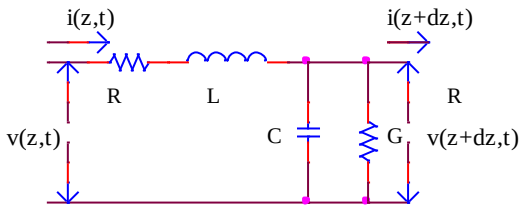

Fig.2 Equivalent circuit model of differential transmission line of length dz.

For a lossless case, the resistance and conductance are zero. The finite length transmission line is obtained by cascading multiple sections of each differential Tx line elements as shown in Fig.3. Here inductor and capacitor are distributed with units henries/meter and farads/meter.

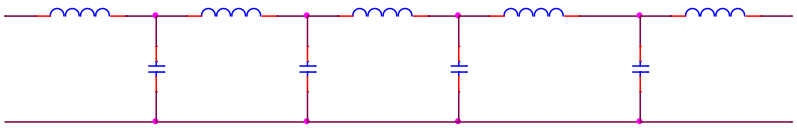

Fig.3 Conventional lossless transmission line model with inductor and capacitor.

Non-linear transmission line theory [5] is an extension of the linear transmission line theory. In non-linear transmission lines, the shunt branch element is replaced with a non-linear element such as varactor diode [6] as shown in Fig.4. These non-linear transmission lines are used in different application in microwave engineering [7].

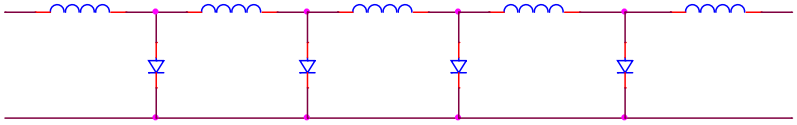

Fig.4 Non-linear transmission line model with varactor diodes.

To analyse the non-linear tranmssion line model, the varactor diode is replaced with the equaivalent voltage dependent capacitor as shown in the Fig.5

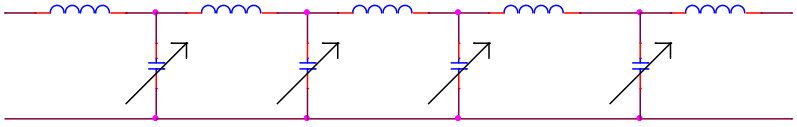

Fig.5 Equivalent circuit model of non-linear transmission line with variable capacitor.

Two most important parameters of the Tx line models are the complex propagation constant and the characteristic impedance. In the analysis of the transmission line model, a unit cell is considered and the wave propagation on a transmission line can be expressed by complex propagation constant.

Complex propagation constant:

Complex propagation constant specifies the medium properties in terms of actuation constant (real part) and phase constant (imaginary part).Depending on the medium, the complex propagation constant is independent of frequency or dependent on frequency. This represents whether the medium actually supports the wave propagation through it or not.

2. Problem formulation:

Electromagnetic waves get distorted while travelling through the medium, if the medium is dispersive [8]. For a dispersive medium the phase constant is a non-linear function of frequency and it is a complicated phenomenon. A typical dispersion characteristic for dispersive medium is shown in Fig.6. In a real transmission line which exhibits the dispersion, the velocity of propagation varies with frequency. The dispersion can be expressed in terms of phase velocity and group velocity.

$$v_p = \omega/\beta \qquad (1)$$

$$v_g = d\omega/d\beta \qquad (2)$$

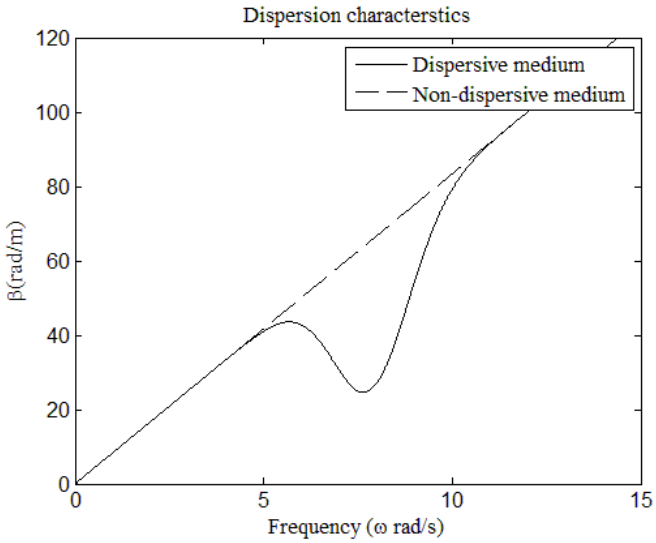

Fig.6 Dispersion characteristics for a dispersive medium and non-dispersive medium.

The phase velocity represents the velocity of the constant phase wavefront while group velocity represents the velocity of the envelope of the pulse. Due to dispersion different frequency components travel with different velocities and if a pulse is travelling in that medium it distorts while it is travelling in the medium. If the dispersion is more, then the pulse will be distorted completely and it cannot be distinguished clearly. This is the disadvantage is data transmission. Even though the dispersion is a negative factor in data transmission, dispersion can also help in pulse shaping circuits. But this dispersion can be a useful factor in some applications such as maintaining the same phase constant at two different frequencies as same. This is used in antenna arrays in beam-steering [9].

Objective: 1

Modeling of the flux dependent inductor as current dependent inductor and charge dependent capacitor as voltage dependent capacitor.

Objective: 2

To analyze the unit cell of the different transmission line model with non-linear elements to get the opposite phase velocity and group velocity.

Objective: 3

To present a simple circuit model which gives opposite phase velocities and group velocities with minimum number of elements in it.

3. Analysis:

In order to meet the condition of opposite phase velocity and group velocity, the analysis on the transmission line model considers with the elements as non-linear. In the literature, the non-linearity is mainly considered with only one element which is a non-linear capacitor that can be realized using varactor diodes. This analysis includes the inductor as non-linear that can be realized from iron core inductor. This analysis also considers both the elements as non-linear which is the other differentiating factor from the analysis by others. The literature only deals with the conventional transmission line model where as this analysis includes three more possibilities to meet the requirement.

3.1 Non-linear inductor:

For an inductor on iron core [10], the relationship between magnetic flux and current passing through the loop are related as

$$i(t) = \frac{N\phi(t)}{L} + A\phi^3(t) \qquad (3)$$

where N is the number of turns of the coil, A is the cross sectional area of the core and L is the self inductance. For a linear inductor the coefficient $A = 0$. Using the Faraday's law of induction along with Lenz's law, the induced voltage can be found out as

$$v(t) = -N\frac{d\phi(t)}{dt} \qquad (4)$$

For a linear inductor the relation between induced voltage and current are related as

$$v(t) = L\frac{di}{dt} \qquad (5)$$

For a non-linear device, it is little difficult to come up with a simple equation of $\phi(t)$ in terms of $i(t)$ not involving radical signs. If the values of A, N and L are known, then with the

help of curve fitting it is easy to invert the same function and express $\phi(t)$ as some function of $i(t)$.

All physical devices exhibit non-linearity. If the modelling of the iron core inductor is more accurate then it is possible to come up with the relationship between flux and current in the higher order terms also. But in practice it is preferred to take the first few order terms in the non-linear expression.

The non-linear relationship between the current and flux assumed in the analysis is represented in Eqn.6.

$$i(t) = a\phi(t) + b\phi^3(t) + c\phi^5(t) \quad (6)$$

with $a = 1, b = 0.01$ and $c = 0.0001$.

Here these values are chosen to see the effect of relatively small non-linearity on the dispersion characteristics of the unit cell model in the transmission line model. Here only Odd order non-linearity is considered due to the fact that the polarity of voltage has to be reversed if the current direction is reversed. This does not happened and hence even non-linearity has been excluded from the relationship to match with the practical devices.

To get the relation between current and voltage for the non-linear capacitor, it is required to express charge as a function of voltage. Based on the numerical procedure of curve fitting with the fifth order polynomial, the Eqn. 6 has been converted into the following equation. The Eqn.7 is an approximation of Eqn.6 and due to the polynomial fit, the constant term with order -16 is present.

$$\varphi = -9.981 \times 10^{-16} + 0.862i - 1.059 \times 10^{-17} i^2 - 0.002317 i^3 + 1.382 \times 10^{-19} i^4 + 3.209 \times 10^{-6} i^5 \quad (7)$$

Using the Faraday's law of induction, the induced potential can be obtained as

$$v(t) = \frac{d\phi}{dt} = \dot{\phi} = L(i)\frac{di(t)}{dt} \quad (8)$$

From this it is clear that the non-linear inductor is acting like a variable inductor which is a function of current passing through the coil. Similar type of assumption can be made for the non-linear capacitor as well.

3.2 Non-linear capacitor:

As there exist a non-linear relationship between the flux and current for an inductor, there exist a non-linear relationship between the charge stored and the potential difference between the plates. In electronics, non-linear capacitor is realized by active devices such as varactor diode in

reverse biased condition [11], transistors and MOS varactor diodes [12]. The non-linearity present in the device varies from one device to another device and the analysis is specific to the type of non-linearity.

The non-linear relationship between charge and voltage stored on the capacitor can be considered as

$$v = aq + bq^3 + cq^5 \qquad (9)$$

The relation between voltage and current can be obtained again by expressing charge (q) as a function of voltage (v).

$$i(t) = \frac{dq}{dt} = \dot{q} = C(v)\frac{di(v)}{dt} \qquad (10)$$

These non-linear equations become the governing equations for the elements.

3.3 Analysis on the unit cell of the transmission line model to get the dispersion characteristic:

The unit cell of the transmission line model contains two inductors and one capacitor. When there are multiple non-linear elements present in the circuit, the equations KCL and KVL become coupled higher order non-linear differential equations. These coupled non-linear differential equations are converted in to first order differential equations with the help of state-space representation [13]. Then the system of equations obtained is solved numerically with the help of fourth order Runge-Kutta method [14] to get the actual currents and voltages across each and every element in the time domain. Then these equations are converted in to the frequency domain using FFT [15] to get the scattering matrices. The algorithm specified in [16] has been used to get the complex propagation constant. Matlab [17] is used for simulations.

4. Simulations and discussions:

In the analysis part, four different circuit models are used.

1. Lowpass equivalent circuit model: Three different cases have been considered for this model.
    a. Shunt capacitor is non-linear.
    b. Series inductors are non-linear.
    c. Both series inductor and shunt inductors are non-linear.
2. Highpass equivalent circuit model with shunt branch inductor as non-linear.
3. Low-pass equivalent model with Miller loading.
4. High-pass equivalent model with Miller loading.

4.1 Lowpass equivalent circuit model.

4.1.1 Shunt capacitor is non-linear:

The first model analyzed is shown in Fig.7. For this the values of the series inductor are taken as 0.01H and the capacitor is non-linear whose constitutive relation is given by

$$v(t) = aq(t) + bq^3(t) + cq^5(t) \qquad (11)$$

with $a = 10, b = 1.0$ and $c = 0.02$.

$$q = -9.133 \times 10^{-16} + 0.08105v + 5.273 \times 10^{-18}v^2 - 1.574 \times 10^{-5}v^3 - 9.401 \times 10^{-22}v^4 + 1.456 \times 10^{-9}v^5 \qquad (12)$$

The values chosen here can be scaled up or down according to the requirement. The choice of these vales does not change the conclusions coming from the analysis.

This structure is exited with voltage in the form of Gaussian pulse

$$v(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{t-\mu}{\sigma})^2} \qquad (13)$$

with $\sigma = 0.05$ and $\mu = 0.25$.

Fig.8 represents the complex propagation constant for the unit cell. The complex propagation constant obtained with the non-linear case has been compared with the linear case to differentiate between the two cases.

From Fig.8, it is clear that relatively small non-linearity is producing variations in the phase constant as a function of frequency which produces the dispersion. From Fig.8 it is observed that around the frequency 25Hz, the propagation constant is changing its slope from negative to positive which gives the opposite phase velocity and group velocity. In Fig. 8 constant β part represents the non-propagating behavior of the Tx line model.

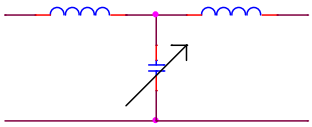

Fig.7 Unit cell of the Tx line model with non-linear capacitor.

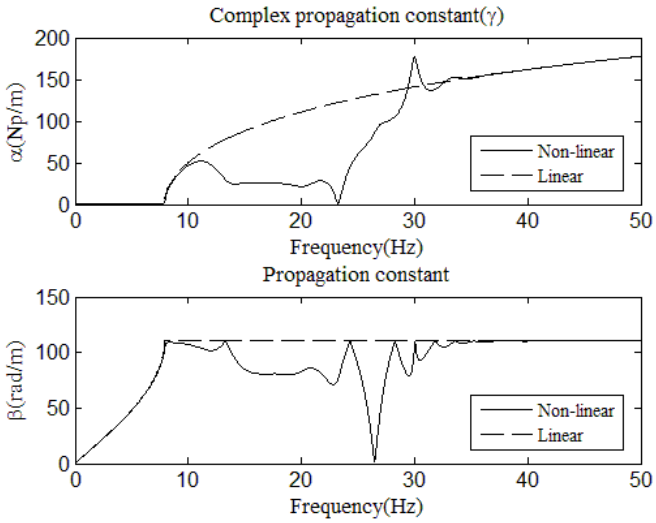

Fig.8 The complex propagation constant for the unit cell with non-linear capacitor.

The reason for the variations in the phase constant has been observe by getting the equivalent shunt impedance of the non-linear capacitor.Fig.9 represents the shunt branch impedance of the considered model. From Fig.9 it is observed that the negative resistance is being produced which is oscillating over the band of frequencies. Due to this the complex propagation constant is deviating from the linear case.

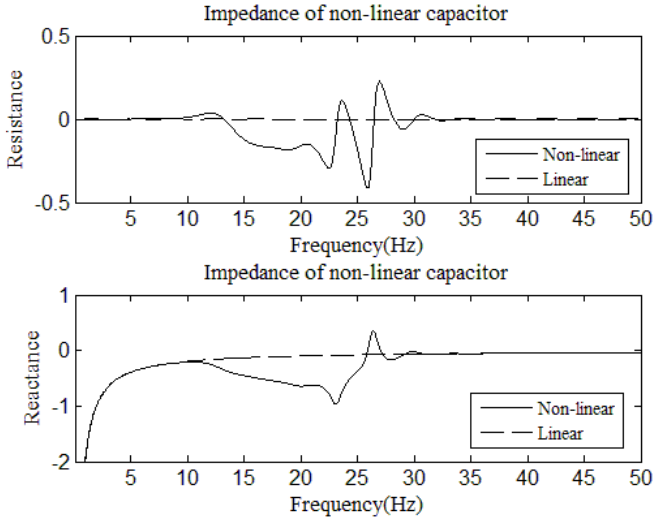

Fig.9 The variation of impedance of the non-linear branch showing the negative resistance.

Along with these observations, the voltage gain of the unit cell is obtained, Fig.10 represents the input voltage applied pulse and output voltage pulse which is almost same as that of the excitation. From this, it can be observed that, very small non-linearity is small dispersion but it is affecting the gain in the stop band of the linear operating region.

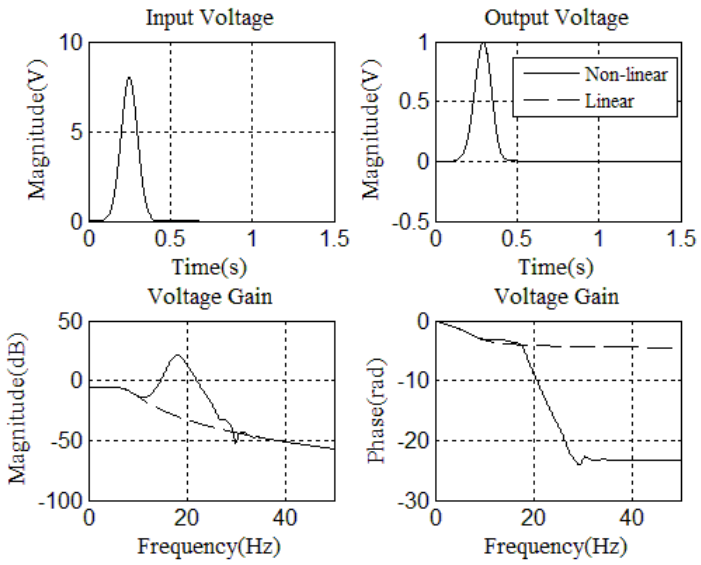

Fig.10 Voltage gain of the unit cell.

The scattering matrices for this unit cell are shown in Fig.11. From this it is clear that there is a non-monotonic variation in the phases of S_{11} and S_{12} which gives the dispersion differently over different frequencies. From this analysis, it is concluded that the relatively small non-linearity can be used in getting the opposite phase velocity and group velocity over a band of frequencies.

But the main requirement is to get the opposite phase velocity and group velocity its smooth transitions while maintaining the attenuation constant small over a band of frequencies. The main dis-advantage with this model is an abrupt variation in the complex propagation constant. To get the smooth variations in the propagation constant, other elements are also considered as non-linear and detailed analysis is carried out.

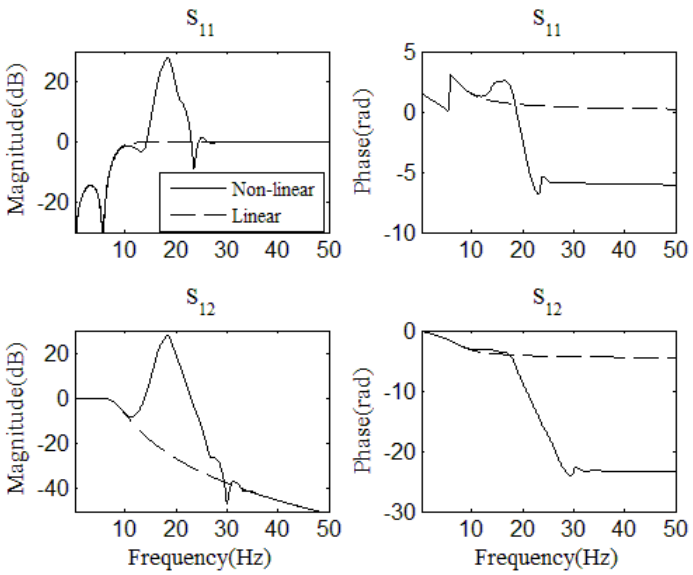

Fig.11 Scattering matrices of the unit cell.

4.1.2 Series inductors are non-linear:

In the second case, the series branch elements have been taken as non-linear elements are shown in Fig.12. The values taken here are $C = 0.01F$ and inductor relationship as $i(t) = a\phi(t) + b\phi^3(t) + c\phi^5(t)$ with $a = 10, b = 1.0$ and $c = 0.02$(This same non-linear relationship is used for the following cases as well). From Fig.13 it is observed that this model is giving smoother variations in propagation constant (β) compared with the previous case which is a slight improvement.

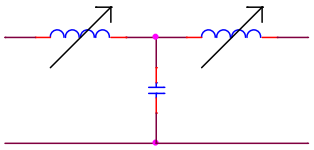

Fig.12 Unit cell of Tx line model with non-linear inductors.

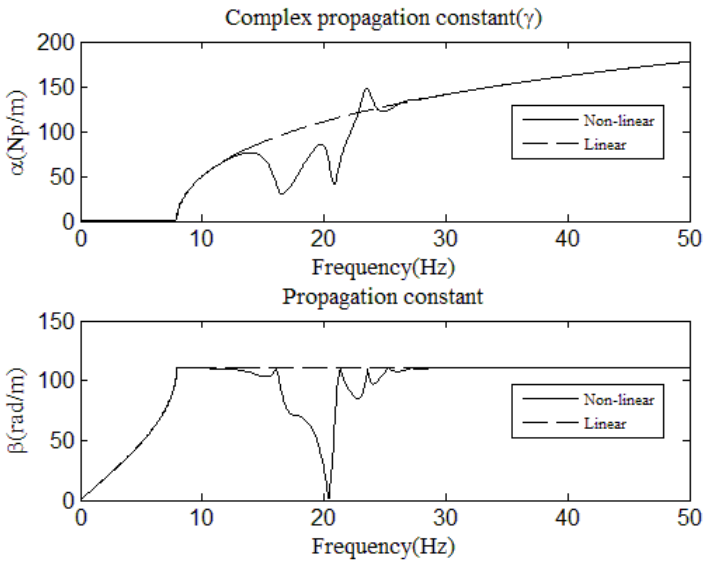

Fig.13 Complex propagation constant.

4.1.3 Both series inductor and shunt inductors are non-linear:

In the final case for the lowpass model, both the elements have been considered as non-linear with same non-linearities as in the previous cases and the propagation constant is much smoother compared with the previous cases as shown in Fig.14. From Fig.15, it is observed that there is a smooth variation in slope of β from 15Hz to 25Hz which is a smooth variation. This model can is better choice for generating opposite phase and group velocities.

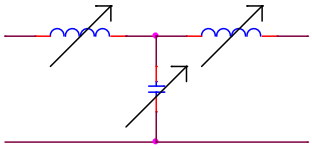

Fig.14 Unit cell of Tx line model with both elements as non-linear.

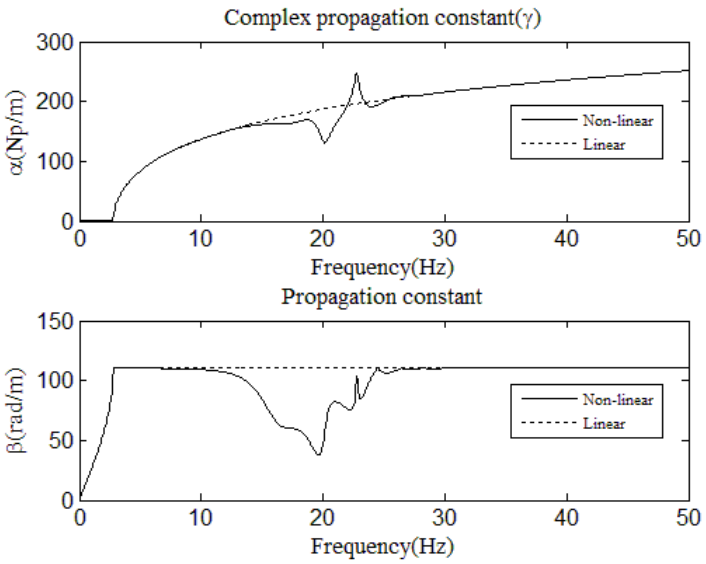

Fig.15 Complex propagation constant.

4.2 Highpass equivalent circuit model with shunt branch inductor as non-linear.

The dual of the previous model has been analyzed with the inductor as non-linear in this case as shown in Fig.16.

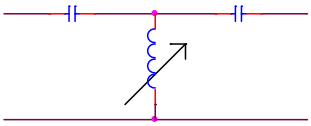

Fig.16 Unit cell of Tx line model with highpass equivalent circuit model.

Fig.17 represents the propagation constant for the high pass circuit model. This is actually showing the opposite phase velocity and group velocity for the entire propagation region. It is again monotonic variation. Due to the non-linearity, there is deviation from the monotonic to non-monotonic which again can be used in specific applications.

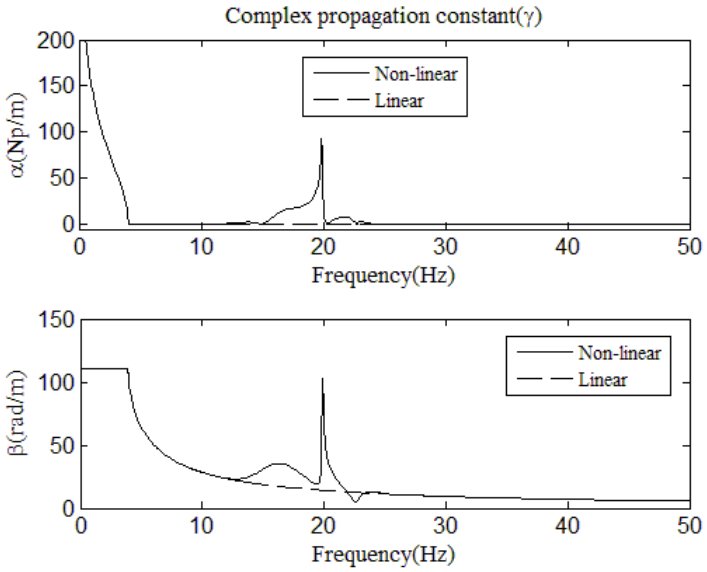

Fig.17 Complex propagation constant.

4.3 Low-pass equivalent model with Miller loading.

Further to implement these devices in the microstrip version with less difficulty, the Miller loading is considered with the element connecting output and input as a non-linear element. Fig.18 represents the considered model for the analysis. This circuit model has been analyzed in infinite transmission line model to get the complex propagation constant for different values of Miller capacitance as a linear element. From these simulations, it is observed that the phase reversal is happening very smoothly for some values of capacitance which is the main advantageous factor. But this approximation is valid if this structure is of infinite line length. Fig.19 represents the complex propagation constant (with non-linear capacitor) which has very smooth variation while maintaining no attenuation with phase reversal.

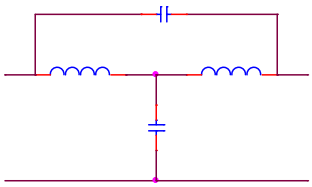

Fig.18 Lowpass equivalent circuit model with Miller loading.

After this, the non-linear capacitor Miller loading of lowpass equivalent model is analyzed. The values taken in this circuit are $L = 0.01H, C = 0.01F$ with C_1(Miller capacitance) as a variable ($q(t) = av(t) + bv^3(t) + cv^5(t)$ with $a = 10$, $b = 1.0$ and $c = 0.02$ is the relationship for non-linear capacitor).

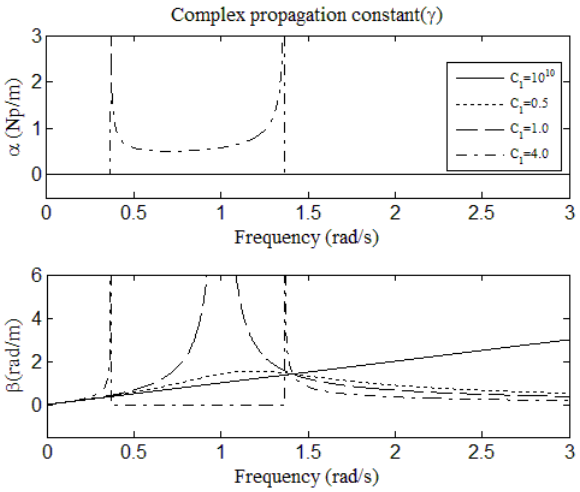

Fig.19 Complex propagation constant

For this structure, the scattering matrices are obtained and from Fig.20, it is observed that phase variations in S_{11} and $S_{12}$with gain more than unity. Here also the phase variations are non-monotonic which produces opposite phase and group velocities.

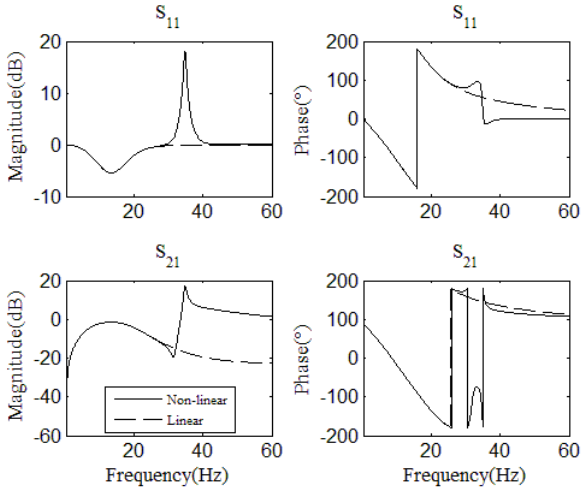

Fig.20 Scattering matrices of lowpass equivalent circuit model with Miller loading.

4.4 Highpass equivalent circuit model with Miller loading.

Finally, the highpass equivalent circuit model with inductive loading is considered. Fig.21 represents the scattering matrices of this model. From Fig.22 it is observed that the phase of S_{21} is varying smoothly in non-monotonic manner while maintain the magnitude of S_{21} above-20dB. The values taken for this circuit model are same as the previous case, with interchanging the capacitance with inductance and vice-versa. For this circuit model, even though gain is less but the change in the gain is smooth which can be controlled with an

amplifier. Hence this model is better choice compared with the lowpass and high pass circuit models with non-linear elements.

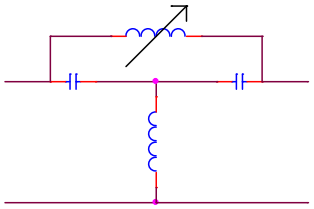

Fig.21 Highpass equivalent circuit model with Miller loading.

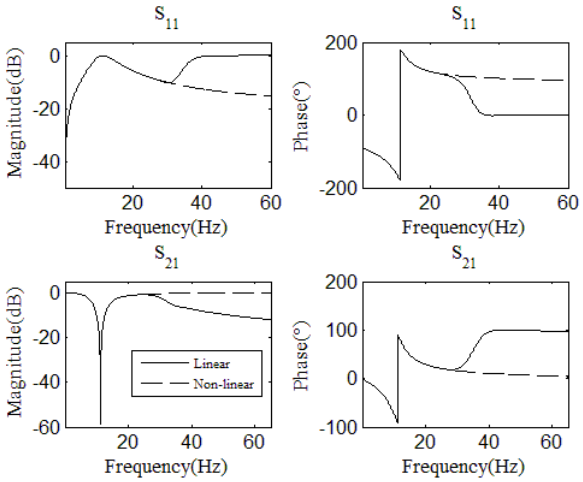

Fig.22 Scattering matrices for highpass equivalent circuit model with Miller loading.

5. Conclusions and future scope:

In this report, the effect of non-linearity on the complex propagation constant of different transmission line models has been analyzed in detail to get the opposite phase and group velocities. This report has considered the relatively small non-linearity (with odd order non-linearity) which is close to the realistic applications. It is also proposed a simpler circuit to have the opposite phase and group velocities with linear elements as well as non-linear elements in the form of Miller loading.

The main disadvantage with the non-linear devices is instability. They are prone to oscillations. Even though there are many uses with the non-linearity, it is difficult to model the practical device in terms of non-linear constitutive relationship. In the future work, the inter-digital capacitor is modeled as non-linear device and these will be implemented in microstrip realizations.

Publications:

[1] Salman Raju Talluri, Sunil Vidya Bhooshan, "Effect of a relatively small non-linear capacitor in the unit cell of transmission line model on scattering parameters and propagation constant " *International Journal of Applied Engineering Research* ,Vol.11, no.16,pp. 3795-3798 ,March,2016.

[2] Salman Raju Talluri, "Delay properties of a transmission line model with non linear Miller loading" *International Journal of Applied Engineering Research* ,Vol.10, no.14,pp. 34985-34988 ,July 2015.

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